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ABSTRACT

Tracing the history of algebra within the U.S. secondary curriculum requires at the same time a review of the various reform movements relevant to secondary education, an acknowledgement of the sociological, political, and psychological perspectives driving these reforms, and most importantly, the perceived role of mathematics in the education of the junior high and high school populations. Making algebra more relevant and developing greater understanding, as is currently being urged, is not a new curriculum concept. What will differ is the learning theory used to achieve understanding. The development of these ideas is carefully traced through the recommendations of influential committees, text materials, and the writings of psychological theorists, all considered in relation to other events of their times. The paper concludes, "just as in the past, skills and structural understanding are the goals of the current curriculum movement. The traditional algebra curriculum works for routine and repetitive problems but not for the development of free and creative thinking so fundamental to mathematical modeling and problem solving. It is the technological advances which will perhaps give us the motivation to stress both conceptual knowledge as well as technical facility in routine skills..." (Author/MNS)

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University of Chicago
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The History of Algebra in the U.S. Secondary Curriculum 1894-1988

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By
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Chicago, Illinois
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The History of Algebra in the U.S. Secondary Curriculum
1894-1988

To adequately present the history of algebra within the U.S. secondary curriculum from 1894 to the present requires at the same time a review of the various reform movements relevant to secondary education, an acknowledgement of the sociological, political, and psychological perspectives driving these reforms, and most importantly, the perceived role of mathematics in the education of the junior high and high school populations. As one of the components of the traditional secondary mathematics curriculum, algebra reflects the changing attitudes of professional educators and the general public alike evolving from a topic of primarily mental disciplinary value to one in which mathematical generalizations are made and applied to realistic problems. Indeed, the current draft of the NCTM's "Curriculum and Evaluation Standards for School Mathematics" clearly defines the purpose of algebraic instruction in contemporary society:

> Algebra is the language through which most of mathematics is communicated. It also provides a means of operating with concepts at an abstract level and then applying them, often in the process permitting generalizations and insights beyond the original context. All citizens of our technologically oriented society will need facility with algebraic processing.[1]

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[1]National Council of Teachers of Mathematics, "Curriculum and Evaluation Standards for School Mathematics" (draft) (Reston, Va.: NCTM, 1987) 103, photocopied.

However, this utilitarian approach to the algebra curriculum, this new and improved algebra more relevant and less prone to pencil and paper manipulations, is not a new curriculum concept. An historical analysis of the various stages of development and implementation of the algebra curriculum reveals a similar focus on understanding of fundamental concepts and their applications. What will differ is the learning theory used to achieve this understanding as well as the very definition of <u>understanding</u> in the algebraic context. Is the understanding generalized and applicable to common, complex, and open problem situations? Or is the understanding compartmentalized, that is, applicable to only categories of problems and not easily transferable (if at all) to analogous problems with similar mathematical structure?

The answers to these questions lie in the formation and interaction of curriculum ideologies voiced at the end of the nineteenth and throughout the twentieth centuries. Initially, the subcommittee for mathematics of the Committee of Ten, known as the Conference on Mathematics, made innovative recommendations regarding a unified approach to the teaching of geometry and algebra. Although the Committee's final report, published in 1894, was mental disciplinarian in nature, it supported the introduction of basic algebraic concepts in the earlier grades to better facilitate a conceptual understanding in later grades. Despite the multitude of references made to this report in subsequent years by the educational community, most high schools of the time did not follow the recommendation for integrating

geometry and algebra, instead forcing completion of algebra before geometry and thus compartmentalizing their respective concepts rather than integrating them.

Textbooks of the period 1894-1928 evidence this trend toward the fragmentation of knowledge acquisition. The text materials focused on algebraic manipulations as isolated skills to be mastered disassociated from the needs and interests of the student. The majority of the instruction dealt with factoring, roots, powers, and basic techniques such as the combination and simplification of rational expressions with little emphasis on functions and their graphical representations, equation solutions, or application problems. Drill and practice with these techniques occupied an estimated 64% of the algebra curriculum during these years with little note of unifying concepts.[2]

Strengthening the isolation of one content area from another, indeed the isolation of one skill or concept from another, was the psychological theory of Edward L. Thorndike and the social efficiency curriculum reform movement. Thorndike's theory, labeled connectionism, emphasized the importance of drill and practice of routine algebraic exercises and, contrary to relating various algebraic concepts, advised against their

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[2]F. Joe Crosswhite and Alan R. Osborne, "Mathematics Education on the Defensive: 1920-45," Thirty-Second Yearbook of the National Council of Teachers of Mathematics: A History of Mathematics Education in the United States and Canada (Washington D.C.: NCTM, 1970), 158-160.

instruction within a close timeframe (Psychology of Algebra, 1923).[3] Similarly, the social efficiency reformers, first led by Joseph Mayer Rice as early as the 1890s and by Edward A. Ross and John Franklin Bobbitt in the two decades following, attuned the curriculum to a new industrial society with its dependence upon repetitive operations both in the factory and classroom. With existing subjects, including algebra, infused with criteria from vocational education, algebraic techniques became tools for trade furthering the isolation of a given technique from the overall concept of algebraic structure.

To counter the monotony and repetitive nature of the algebra curriculum continuing into the 1920s, The 1923 Report listed as one of its practical aims "...an understanding of the language of algebra and the ability to use this language intelligently and readily."[4] The Committee attacked the psychological theory of Thorndike and the sociological perspective of Ross and Bobbitt advocating instead the application of the fundamentals of algebra in a generalized framework. More specifically, the educational leaders of the time wanted a more streamlined curriculum different from the older by an elimination of approximately one quarter of the content (e.g., contrived problems outside the

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[3]Arthur F. Coxford, Jr. and Phillip S. Jones, "Mathematics in the Evolving Schools," Thirty-Second Yearbook of the National Council of Teachers of Mathematics: A History of Mathematics Education in the United States and Canada (Washington D.C.: NCTM, 1970), 38.

[4]Ibid., 203.

realm of science, business, and industrial applications) and an emphasis on the mental construction of algebraic relationships. Unfortunately, these recommendations for algebra reform were well cited but were only occasionally implemented in texts during the 1920s and 1930s. Though The 1923 Report and C.E.E.B. reports were often mentioned in textbook prefaces, the recurring emphasis still remained algebraic manipulation with the primary method of instruction following a definition, illustration, rule and example, drill, review, and speed test sequence.

An exception to this methodology for algebra instruction were the texts written during the late 1920s and early 1930s (as well as decades following) by William Betz, vice-principal of East High School and Specialist in Mathematics for the public schools in Rochester, New York. Betz organized algebra content around the concept of function seeking to extend the concept beyond its traditional mathematical context to a higher level of functional thinking. Criticizing the imitative, mechanical style of learning, Betz described algebra as laden with the "curse of empty symbolism" countering the unification of objectives:

> A symbol arises when I use a word, or a written sign, to refer to a concept...This suggests that if the concept for which the symbol is supposed to stand does not correspond to anything that has been experienced, the symbol has no meaning. But, a symbol which does not symbolize anything is useless.[5]

Yet, in 1934 when Betz made reference to this dilemma in the Mathematics Teacher, classroom teachers were basing algebraic

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[5]William Betz, "Whither Algebra?--A Challenge and a Plea, "Mathematics Teacher 23 (January 1930): 114.

technique on unrelated rules too numerous to be memorized and appearing in too many combinations for practical use. Although Betz recognized the importance of the mastery of essential skills prior to use in problem solving, he authored innovative texts and proposed thinking in terms of interrelated ideas instead of meaningless symbols.

With the depression years and overall increase in school attendance due to unemployment, a practical approach to algebra instruction was implemented. The shift to life-situation problems with corresponding illustrations assisted this change. Concepts were introduced around important centers of interest for the student. For instance, textbook authors devoted entire units to the application of formulas and relations to money, the shop, and science; the clear emphasis was on clear thinking with everyday applications.

This same trend continued throughout the 1940s and early 1950s with major reports published by the Commission on Postwar Plans and the Joint Commission. Both groups espoused functional competence as the primary goal of algebra instruction but with the report of the Joint Commission highlighting as well the importance of dependence through tables, graphs, and formulas. The decade of the fifties became a decade of change, however, as several curriculum projects were undertaken during this era beginning with the University of Illinois Committee on School Mathematics in 1952, the University of Maryland Mathematics Project in 1957, and the School Mathematics Study Group in 1958.

Also influential was the C.E.E.B. and its 1959 report on the modernization of the secondary mathematics curriculum.

Since many features of these curriculum reform projects coincide, it is more appropriate to discuss the similarities in the newly developed algebra curricula. Although the C.E.E.B. did not publish a ninth grade algebra text, its recommendations coincided with the features of the corresponding U.I.C.S.M. and S.M.S.G. texts. Specifically, the groups stressed the importance of deductive reasoning, the appreciation of mathematical structure illustrated by properties of sets of numbers, the use of integrating concepts such as sets, variables, functions, and relations and the study of inequalities in conjunction with equations. Of particular note was the creation of the word pronumeral by the U.I.C.S.M. to facilitate the introduction of the concept of numerical variable.[6] The Maryland Project also considered structural understanding developed through active student participation as its main objective restricting its efforts to seventh and eighth grade needs.

Following the completion of the projects described, especially during the sixties, the student learned basic ideas through a more theoretical approach involving Venn diagrams and the union and intersection of sets. This same theoretically-based algebra curriculum geared toward the better student continued into the seventies as most of the texts used were those

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[6]Herbert Fremont, Teaching Secondary Mathematics (Boston: Prindle, Weber and Schmidt, 1979), 42-43.

written during the sixties.[7] Many texts did, however, insert basic ideas as to flowcharting and input/output. Practical use of these concepts was not prevalent. The next wave of influence, the back-to-basics movement, advocated a return to the drill and practice exercises of two decades earlier. Thus came an abandonment of theory presumably in response to the needs of the average student, perhaps a predictable response in light of the cyclical trends in algebra reform described earlier.

Immediately thereafter, national reports, first from the National Advisory Committee on Mathematical Education (NACOME) and the National Institute of Education in 1975 and later from the National Council of Supervisors of Mathematics in 1976 and the NCTM in its An Agenda for Action in 1980, stressed the duality of purpose of the secondary mathematics curriculum, a duality focusing concurrently on computational skill (instrumental understanding) and relational understanding (Skemp, 1987). More importantly for true innovation in the algebra curriculum, all these reports promoted the use of calculators and computers as an alternative to tedious drill in the classroom. This philosophy continues today in the eighties and is best summarized perhaps by James Fey of the University of Maryland in his analysis of algebra and computing:

> If computers can do algebraic manipulations like factoring, solving equations...operating on matrices,

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[7]Zalman Usiskin, "We Need Another Revolution in Secondary Mathematics," in 1985 Yearbook of the National Council of Teachers of Mathematics, The Secondary School Mathematics Curriculum (Reston, Va.: NCTM, 1985), 7.

> the importance of developing student skill in those areas is seriously questioned...All this is available now on microcomputers at moderate cost but soon we can expect the power in...inexpensive hand-held hardware...At the very least we should be able to identify a level of performance that is more efficiently done by humans and to consign the more complex cases to machine execution.[8]

What Fey proposes is not a complete neglect of the mastery of basic skills. What he does propose is the student's awareness of different algebraic forms and the information best extracted from each. If computer software such as muMath can perform the more traditional exercises of simplification and quadratic equation solution, according to Fey, students may begin with more interesting and relevant application problems culminating in the recognition and mathematization of relations, answering questions about functions, and formal organization and verification of the more basic principles discovered through computer technology.[9]

Current research and publications concerning the algebra curriculum and instruction continue to support the need for a more streamlined, conceptually-based curriculum content. The University of Wisconsin Center for Education Research and its School Mathematics Monitoring Project (March, 1987) and The Ideas of Algebra, K-12, 1988 Yearbook from the NCTM offer similar suggestions for the use of computer programming and computer-

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[8]James T. Fey, "Rethinking the Sequence and Priorities of High School Mathematics Curricula," in 1985 Yearbook of the National Council of Teachers of Mathematics, The Secondary School Mathematics Curriculum (Reston, Va.: NCTM, 1985), 7.

[9]Ibid., 49.

generated data to introduce concepts and the use of the calculator as a tool to eliminate repetitive calculations.

It would seem then that, just as in the past, skill and structural understanding are the goals of the current curriculum movement. The traditional algebra curriculum works for routine and repetitive problems but not for the development of free and creative thinking so fundamental to mathematical modeling and problem solving. It is the technological advances which will perhaps give us the motivation to stress both conceptual knowledge as well as technical facility in routine skills as an alternative to our historical cycle of first emphasizing computation, then theory, but never with a consolidated presentation of the two. As has been seen throughout the history of the subject, theoretical knowledge is neither necessary nor sufficient for success in algebra. However, procedural knowledge based upon rote learning is not enough. Likewise, theory without technical facility in applications is not enough. By employing simultaneously the structural consistencies and patterns of algebra and the computational abilities of the calculator and the computer, we may initiate the study of algebra not as a subject of confusing rules but as a subject of structure and consistency. Mathematics education is too important to rely on trends. Given this, future questions must focus on the level of expertise expected of the student in both areas of computation and algebraic analysis and interpretation beyond that provided by technology. In other words, what constitutes a true

understanding of algebra, both in the instrumental and relational senses, once this new and improved algebra is put into place, if it ever is?

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